Computing With Words

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Abstract

- Multi expert multi criteria decision making problems (MEMCDMP) seek to choose the optimal alternative among the finite number of alternatives available based on the evaluations of a set of experts on certain prefixed criteria.
- We assume that the criteria are described by the decision maker in form of linguistic expressions, like, very good, good, fair, bad, worst, etc.
- The idea is to develop the model which can aggregate the expertise of all decision makers in form of linguistic variables to come up with the optimal ranking of finite alternatives.

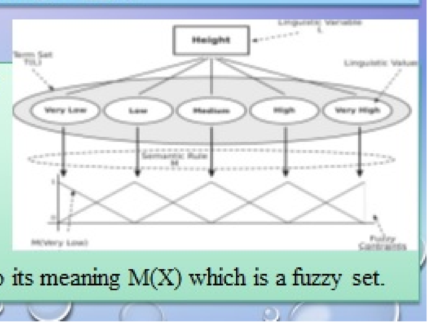
Linguistic variable

CW - Words and propositions are objects of computation like small, medium, big, heavy, light etc.

LINGUISTIC VARIABLE:

- **LINGUISTIC VARIABLE** is a variable whose values are expressed in linguistic terms.
- It's Values are not numbers but words or propositions from natural language.
- Used to deal with qualitative or fuzzy expressions (The Building is **tall**)

- Linguistic variable is a **5 Tuple (L, T(L), U, S, M)** where:
  L – Name of the variable
  T(L) – Term set (A collection of Linguistic values)
  U – Universe of discourse
  S – Syntactic rule to generate terms in T(L)
  M – Semantic rule to associate each linguistic value X to its meaning M(X) which is a fuzzy set.



2 tuple representation of linguistic data [1]

- Let $S = \{s_0, s_1, , s_g\}$ be the linguistic term set of cardinality g+1.
  Let $\beta \in [0, g]$ be the result of the aggregation operation
  Then, 2 tuple equivalent to β is given by:

$$\Delta: [0, g] \rightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha) \quad \text{with} \quad \begin{cases} i = \text{round}(\beta) \\ \alpha = \beta - i \\ \alpha \in [-0.5, 0.5) \end{cases}$$

- Numerical equivalent to (s_i, α) is given by:

$$\Delta^{-1}: S \times [-0.5, 0.5) \rightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Example

EXAMPLE :

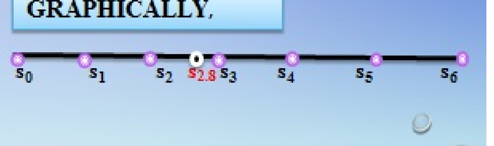
Let
$S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ be the linguistic term set with cardinality 7.

Suppose by using aggregation operator we obtain the result

$\beta = 2.8$
THEN,
$\Delta(\beta) = \Delta(2.8) = (s_i, \alpha)$
$= (s_3, -0.2)$

$$\begin{cases} i = \text{round}(\beta) = \text{round}(2.8) = 3 \\ \alpha = \beta - i = 2.8 - 3 = -0.2 \end{cases}$$

GRAPHICALLY,



Comparision operator

1) COMPARISON OF 2 TUPLES: (LEXICOGRAPHIC ORDER)

LET (s_i, α_1) AND (s_j, α_2) BE TWO 2-TUPLES:

- If $i < j$
  Then $(s_i, \alpha_1) < (s_j, \alpha_2)$
- If $i = j$ and $\alpha_1 = \alpha_2$
  Then $(s_i, \alpha_1) = (s_j, \alpha_2)$
- If $i = j$ and $\alpha_1 < \alpha_2$
  Then $(s_i, \alpha_1) < (s_j, \alpha_2)$
- If $i = j$ and $\alpha_1 > \alpha_2$
  Then $(s_i, \alpha_1) > (s_j, \alpha_2)$

EXAMPLES:

- $(s_3, 0.4) < (s_4, 0.2)$
- $(s_3, 0.4) = (s_3, 0.4)$
- $(s_3, 0.3) < (s_3, 0.4)$
- $(s_3, 0.3) > (s_3, 0.1)$

Arithmetic mean operator

1) 2- TUPLE LINGUISTIC ARITHMETIC MEAN:

Let
$X = \{(s_1, \alpha_1), (s_2, \alpha_2), ..., (s_n, \alpha_n)\}$
be set of linguistic 2 tuples. Then **2 tuple arithmetic mean** is defined as:

$$X_{MEAN} = \Delta\left(\frac{1}{n}\sum_{i=1}^{n}\Delta^{-1}(s_i, \alpha_i)\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^{n}\beta_i\right)$$

EXAMPLE:
Let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$
$X = \{ (s_1, 0.4), (s_3, 0), (s_4, -0.4), (s_4, 0) \}$

$X_{(AM)} = \Delta[(\Delta^{-1}(s_1, 0.4) + \Delta^{-1}(s_3, 0) + \Delta^{-1}(s_4, -0.4) + \Delta^{-1}(s_4, 0)) / 4]$
$= \Delta[(1.4 + 3 + 3.6 + 4) / 4] = \Delta(3) = (s_3, 0)$

Ordered weighted averaging operator

3) 2- TUPLE LINGUISTIC ORDERED WEIGHTED AVERAGE (OWA):

Let $X = \{(s_1, \alpha_1), (s_2, \alpha_2), (s_3, \alpha_3), \ldots, (s_n, \alpha_n)\}$ be set of linguistic 2 tuples.

Each value x_i has a weight associated w_i indicating its importance
But
THE ASSOCIATED WEIGHTS ARE NOT PREDETERMINED BUT ASSOCIATED TO A DETERMINED POSITION.

Let $W = \{w_1, w_2, w_3, \ldots, w_n\}$ be associated weights,
$0 \le w_i \le 1$, $\sum w_i = 1$.

Then **2 TUPLE ORDERED WEIGHTED MEAN** is defined as:

$$X_{OWA} = \Delta\left(\sum_{r=1}^{n} w_r \beta_r'\right)$$

where β_r' is the r th largest value of β_i.

Power averaging operator

(1) 2 TUPLE LINGUISTIC POWER AVERAGING OPERATOR (2TLPA):

Let $\{(r_1, \alpha_1), (r_2, \alpha_2), \ldots, (r_n, \alpha_n)\}$ be the set of linguistic 2 tuples,
Then,

$$2TLPA((r_1, \alpha_1), (r_2, \alpha_2), \ldots, (r_n, \alpha_n)) = \Delta\left(\frac{\sum_1^n (1+T(r_i, \alpha_i))\Delta^{-1}(r_i, \alpha_i)}{\sum_1^n (1+T(r_i, \alpha_i))}\right)$$

where $T(r_i, a_i) = \sum_{j=1, j \ne i}^{n} Sup((r_i, \alpha_i), (r_j, \alpha_j))$

$Sup((r_i, \alpha_i), (r_j, \alpha_j))$ **DENOTES THE SUPPORT OF** $(r_i, \alpha_i), (r_j, \alpha_j)$.

It is any function satisfying the following 3 properties:

Support is similarity index between the linguistic tuples, more the similarity more is the support.

1) $Sup((r_i, \alpha_i), (r_j, \alpha_j)) \in [0,1]$
2) COMMUTATIVE: $Sup((r_i, \alpha_i), (r_j, \alpha_j)) = Sup((r_j, \alpha_j), (r_i, \alpha_i))$
3) $Sup((r_i, \alpha_i), (r_j, \alpha_j)) \le Sup((r_k, \alpha_k), (r_l, \alpha_l))$ if $d((r_i, \alpha_i), (r_j, \alpha_j)) > d((r_k, \alpha_k), (r_l, \alpha_l))$.

Where d is the distance between the linguistic variables.

Ordered weighted power averaging operator

(2) 2 TUPLE LINGUISTIC WEIGHTED POWER AVERAGING OPERATOR (2TLWPA):

Let $\{(r_1, \alpha_1), (r_2, \alpha_2), \ldots, (r_n, \alpha_n)\}$ be the set of linguistic 2 tuples,
Let $W = \{w_1, w_2, w_3, \ldots, w_n\}$ be associated weights,
$0 \le w_i \le 1$, $\sum w_i = 1$.
Then,

$$2TLPWA((r_1, \alpha_1), (r_2, \alpha_2), \ldots, (r_n, \alpha_n)) = \Delta\left(\frac{\sum_1^n (1+T(r_i, \alpha_i))\Delta^{-1}(r_i, \alpha_i) w_i}{\sum_1^n (1+T(r_i, \alpha_i)) w_i}\right)$$

SPECIAL CASE:

- If $W = \{\frac{1}{n}, \ldots \frac{1}{n}\}$, Then 2TLWPA = 2TLPA .

{ When all the Decision makers are of equal importance, then 2TLWPA = 2TLPA. }

Interval 2 tuple linguistic representation

INTERVAL 2 TUPLE LINGUISTIC VARIABLES:

Let $S = \{s_0, s_1, \ldots, s_g\}$ be the linguistic term set.
An **INTERVAL 2 TUPLE LINGUISTIC VARIABLE** composes of two 2 tuples:
Denoted by $[(s_i, \alpha_i), (s_j, \alpha_j)]$, $(s_i, \alpha_i) \le (s_j, \alpha_j)$.

The Interval 2 tuple that expresses equivalent information to Interval value $[\beta_1, \beta_2]$, $\beta_1 \le \beta_2$ is given by:

$$\Delta[\beta_1, \beta_2] = [(s_i, \alpha_i), (s_j, \alpha_j)] \text{ with } \begin{cases} i = \text{round}(\beta_1 . g) \\ j = \text{round}(\beta_2 . g) \\ \alpha_i = \beta_1 - \frac{i}{g}, \ \alpha_i \in [\frac{-0.5}{g}, \frac{0.5}{g}) \\ \alpha_j = \beta_2 - \frac{j}{g}, \ \alpha_j \in [\frac{-0.5}{g}, \frac{0.5}{g}). \end{cases}$$

- Function Δ^{-1} converts interval 2 tuples back to interval value $[\beta_1, \beta_2]$:

$$\Delta^{-1}[(s_i, \alpha_i), (s_j, \alpha_j)] = [\frac{i}{g} + \alpha_i, \frac{j}{g} + \alpha_j] = [\beta_1, \beta_2].$$

REMARK :

If $(s_i, \alpha_i) = (s_j, \alpha_j)$ then the Interval 2 tuple linguistic variable $[(s_i, \alpha_i), (s_j, \alpha_j)]$ reduces to 2 tuple linguistic variable (s_i, α_i).

INTERVAL 2 TUPLE WEIGHTED AVERAGE (I2TWA):

Let $X = \{ [(s_1, \alpha_1), (t_1, \varepsilon_1)], \ldots, [(s_n, \alpha_n), (t_n, \varepsilon_n)] \}$ be the set of interval 2 tuples
$W = \{w_1, w_2, w_3, \ldots, w_n\}$ be associated weights ,
$0 \le w_i \le 1$, $\sum w_i = 1$

Then,
Interval 2 tuple weighted average is defined as:

$$I2TWA(X) = \Delta\left[\sum_{i=1}^{n} w_i \Delta^{-1}(s_i, \alpha_i), \sum_{i=1}^{n} w_i \Delta^{-1}(t_i, \varepsilon_i) \right]$$

DISTANCE BETWEEN 2 INTERVAL 2 TUPLES:

Let $a = [(s_1, \alpha_1), (t_1, \varepsilon_1)]$ and $b = [(s_1', \alpha_1'), (t_1', \varepsilon_1')]$ be 2 interval 2 tuples,
Then,

$$D(a, b) = \Delta\sqrt{([(\Delta^{-1}(s_1, \alpha_1) - \Delta^{-1}(s_1', \alpha_1'))^2 + (\Delta^{-1}(t_1, \varepsilon_1) - \Delta^{-1}(t_1', \varepsilon_1'))^2])}$$

is the **EUCLIDEAN DISTANCE** between a and b.

DISTANCE BETWEEN 2 SETS OF INTERVAL 2 TUPLES:

Let $X_1 = \{ [(s_1, \alpha_1), (t_1, \varepsilon_1)], \ldots, [(s_n, \alpha_n), (t_n, \varepsilon_n)] \}$
$X_2 = \{ [(s_1', \alpha_1'), (t_1', \varepsilon_1')], \ldots, [(s_n', \alpha_n'), (t_n', \varepsilon_n')] \}$ be 2 sets of interval 2 tuples,
Then,

$$D(X_1, X_2) = \Delta\sqrt{(\sum[(\Delta^{-1}(s_i, \alpha_i) - \Delta^{-1}(s_i', \alpha_i'))^2 + (\Delta^{-1}(t_i, \varepsilon_i) - \Delta^{-1}(t_i', \varepsilon_i'))^2])}$$

is the **EUCLIDEAN DISTANCE** between X_1 and X_2.

TOPSIS

FLOW DIAGRAM OF THE INTERVAL 2 TUPLE LINGUISTIC TOPSIS ALGORITHM

Hesitant fuzzy sets [3]

- Let X be a set then Hesitant fuzzy set on X is a function $h : X \longrightarrow P([0, 1])$ that returns a subset of [0,1]. h returns a set of membership functions for each element in domain. When we consider non empty and finite HFS then it is THFS.
- **Dual hesitant fuzzy set**
  Let X be a set then DHFS on X is defined as $D = \{< x, h(x), g(x) > | x \in X\}$ where h(x) and g(x) are 2 sets of values in [0,1] denoting membership and non - membership degrees of element $x \in X$ such that $0 \leqslant \gamma, \eta \leqslant 1$ and $0 \leqslant \gamma^+ + \eta^+ \leqslant 1$ where $\gamma \in h(x), \eta \in g(x)$, $\gamma^+ = Max_{\gamma \in h(x)}\{\gamma\}, \eta^+ = Max_{\eta \in g(x)}\{\eta\}$.
  The pair (h(x), g(x)) is called DHFE.
- **Hesitant fuzzy linguistic term sets (TFLTS)**
  Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set then HFLTS is ordered finite subset of consecutive linguistic terms of S.
  $H_s = \{s_i, s_{i+1}, \ldots s_j\}$ such that $s_k \in S, k \in$ (i to j).

Choquet integral aggregation operators for 2 tuple linguistic data

- Linguistic group decision making problem with interdependent attributes is optimized using Choquet integral based aggregation operators.
- **2TLCA**
  Let $\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\}$ be 2 tuple linguistic arguments, X be the set of attributes, μ be the fuzzy measure on X then the 2 tuple linguistic correlated averaging (2TLCA) operator is defined as follows:

$$2TLCA = \Delta\left(\sum_{i=1}^{n}(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)}) \ \Delta^{-1}(r_{\sigma(i)}, a_{\sigma(i)})\right)$$

  where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of (1,2,...,n) such that $(r_{\sigma(i)}, a_{\sigma(i)}) \ge (r_{\sigma(i+1)}, a_{\sigma(i+1)})$ and $x_{\sigma(i)}$ is the attribute of $(r_{\sigma(i)}, a_{\sigma(i)})$, $H_{\sigma(i)} = \{x_{\sigma(k)} | k \le i\}$ for $i \ge 1$, $H_{\sigma(0)} = \phi$.
- **2TLCG**

$$2TLCG = \Delta\left(\prod_{i=1}^{n}(\Delta^{-1}(r_{\sigma(i)}, a_{\sigma(i)}))^{(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)}))}\right)$$

Induced continuous Choquet integral operators [2]

- If the information about the weights of the experts and the attributes is incompletely known, consistency principle and TOPSIS method are used to develop a model on the fuzzy measures of experts and attributes which is solved to compute the exact weights from the partial interval weights already known for the experts and the attributes.
- If $\lambda = \int_0^1 Q(y)dy$ then COWA $= F_Q([a, b]) = (1 - \lambda)a + \lambda b$ and COWG $= G_Q([a, b]) = a^{1-\lambda}b^{\lambda}$.
- **Induced continuous Choquet ordered weighted averaging operator**
  ICCOWA is defined to aggregate a set of arguments of 2 tuples $\{< u_1, [a_1, b_1] >, \ldots, < u_n, [a_n, b_n] >\}$ as:

$$ICCOWA = \sum_{j=1}^{n}(\mu(A_{\sigma(j)}) - \mu(A_{\sigma(j+1)}))F_Q([a_{\sigma(j)}, b_{\sigma(j)}])$$

  where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of (1,2,...,n) such that $u_{\sigma(j)} \le u_{\sigma(j+1)}$ and $A_{\sigma(j)} = \{[a_{\sigma(k)}, b_{\sigma(k)}] | j \le k \le n\}$, $A_{\sigma(n+1)} = \phi$.
- **Induced continuous Choquet geometric mean operator**
  ICCGM is defined to aggregate a set of arguments of 2 tuples $\{< u_1, [a_1, b_1] >, \ldots, < u_n, [a_n, b_n] >\}$ as:

$$ICCGM = \prod_{j=1}^{n}(G_Q([a_{\sigma(j)}, b_{\sigma(j)}]))^{(\mu(A_{\sigma(j)}) - \mu(A_{\sigma(j+1)}))}$$

Plan

- From research point of view, currently we are working on the theory of and trying to find an appropriate way to solve linguistic linear programming problems, which are not already present in the literature.

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